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THE BUCKLING UNDER PURE BENDING OF A PLATE GIRDER REINFORCED BY MULTIPLE LONGITUDINAL STIFFENERS

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Abstract—The paper provides solutions for the buckling under pure bending of a plate girder web which is reinforced by 3, 4 or 5 longitudinal stiffeners. These solutions are presented for the cases where both of the longitudinal edges are assumed to be either simply supported or rigidly clamped. In both cases it is assumed that a simple support condition exists along the transverse stiffeners.

The optimum positioning of these stiffeners is discussed and numerical relationships between the stiffener parameters γ and B and the aspect ratio α are presented for the case where all of the longitudinal stiffeners are identical.

NOTATION

d	panel depth
b	panel length
t	plate thickness
μ	Poisson's Ratio
D	$= Et^3/12(1-\mu^2)$ flexural rigidity of unit width of plate
I_q	Second moment of area of the qth stiffener
$\gamma_q = \dfrac{EI_q}{Dd}$	ratio of flexural rigidity of the qth longitudinal to the flexural rigidity of the plate
Δ_q	area of the qth longitudinal stiffener
$B_q = \dfrac{\Delta_q}{td}$	ratio of area of the qth longitudinal stiffener to area of panel
α	aspect ratio of panel $= b/d$
σ_{cr}	$= K\pi^2 D/d^2 t$ actual compressive stress at edge of plate
K	non dimensional buckling coefficient
ξ, η	Cartesian co-ordinates
w	transverse deflection of plate's middle surface
η_q	value of η at qth longitudinal stiffener
P	number of longitudinal stiffeners

1. INTRODUCTION

In an earlier paper [1], the authors presented a theoretical solution for the buckling of a web reinforced by any number of longitudinal stiffeners and also provided numerical results for the case of two longitudinal stiffeners. This present paper examines the case of a web reinforced by either 3, 4 or 5 longitudinal stiffeners and provides numerical results for the particular case where all of the stiffeners have the same size.

The theoretical solution assumed that the longitudinal stiffeners were symmetrically placed about the mid-plane of the web and that their torsional rigidity was negligible.

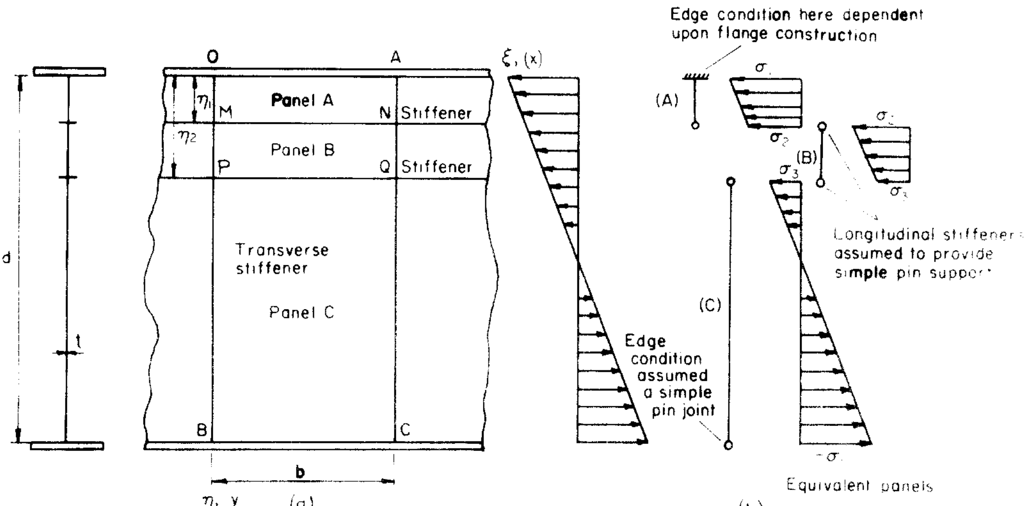

FIG. 1.

The structure considered is shown in Fig. 1, where OB and AC are transverse stiffeners and MN and PQ are typical longitudinal stiffeners. It was assumed that the transverse stiffeners provided a simple support to the panel OBCA along the edges OB and AC. It was further assumed that the longitudinal edges OA and BC were either simply supported or rigidly clamped. By considering both of these edge conditions the influence of any variation in edge restraint upon the buckling strength of the web and the relationships between the parameters γ, B and α can be assessed.

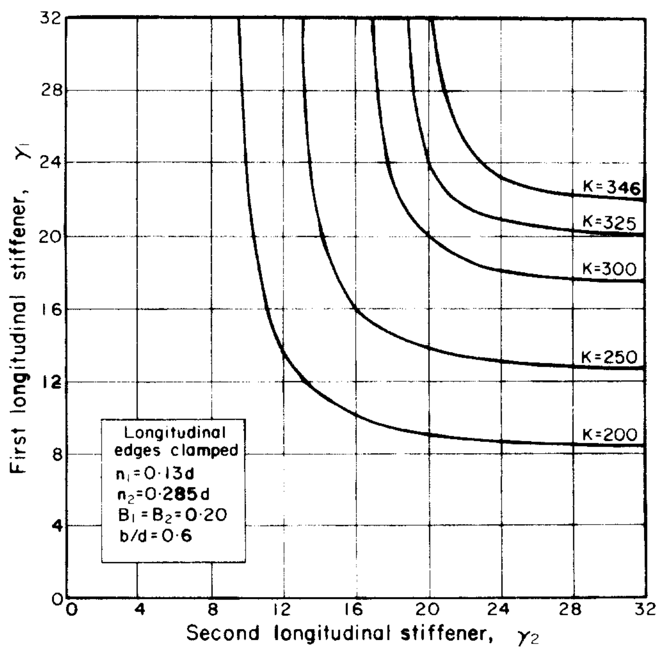

FIG. 2.

The authors also showed that for given values of K, $\eta_1, \eta_2, \ldots, \eta_p$, there is a single relationship between the stiffener parameters, $\gamma_1, \gamma_2, \ldots, \gamma_P$; $B_1, B_2, \ldots, B_P$, and the aspect ratio α. In the case of P stiffeners this relationship is linear in each γ_i and involves $(2P-1)$ independent coefficients for each value of α. The manner in which a change in the rigidity of one stiffener can affect the rigidity of the other stiffeners is shown in Fig. 2, which gives the relationships between γ_1, γ_2 and K for the case of two longitudinal stiffeners only. When there are more than two stiffeners, the relationship between $\gamma_1, \gamma_2, \ldots \gamma_P$ for any given value of K is more complicated. In practice it is most convenient when all the stiffeners have the same size and the numerical results presented in this paper are for this particular case.

2. DETERMINATION OF OPTIMUM STIFFENER POSITIONS

The determination of the optimum stiffener positions for the case of only two stiffeners is not a simple matter. Figure 2 of Reference 1 shows how the value of K varies with the positioning of the two stiffeners when the longitudinal edges are clamped. In this particular case the maximum value of K which can be obtained is 356 when the two stiffeners are at $0{\cdot}136d$ and $0{\cdot}284d$ from the compression flange. Clearly, the determination of the optimum stiffener positions is very much more difficult when more than two stiffeners are employed and the development of an approximate method of determining their positions is desirable.

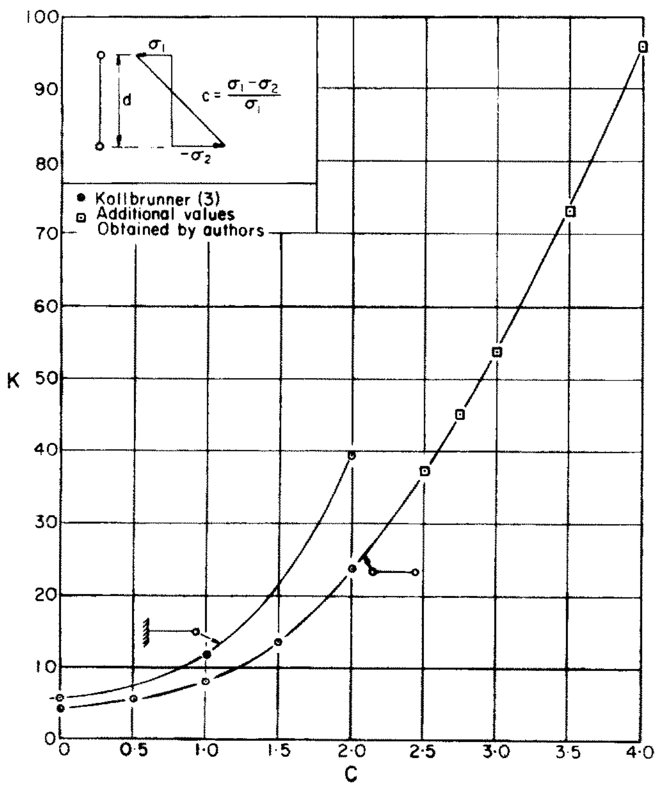

FIG. 3.

Figure 1 shows the elevation of a plate girder reinforced by two stiffeners. The buckling stress of the various panels can be computed treating them as individual rectangular panels, assuming that the stiffeners and adjacent panels provide a simple support. Clearly this would be a lower bound to the actual buckling stress since it neglects any continuity effect. Kollbrunner [3] has collected together the available solutions which deal with the buckling of rectangular panels under linearly varying edge stress; these are plotted in Fig. 3. Further values of K have been determined by the writers, as indicated in Fig. 3.

For the case where both edges of the panels are simply supported, the following empirical laws between K and C have been determined:

$$K = 2{\cdot}2C^3 + 1{\cdot}61C^{\frac{1}{2}} + 4 \qquad \text{Valid } C \leq 2, \tag{1a}$$

$$K = 5{\cdot}98C^2 \qquad C \geq 2. \tag{1b}$$

When the edge of a rectangular panel which is subjected to the greatest compressive stress is clamped and the other edge is simply supported, the relationship between K and C is given by equation (2):

$$K = 3{\cdot}97C^3 + 2{\cdot}35C + 5{\cdot}41. \tag{2}$$

Now, considering panel A of Fig. 1, the buckling coefficient K for this panel is

$$K_a = \left[3{\cdot}97\left(\frac{2\eta_1}{d}\right)^3 + 2{\cdot}35\left(\frac{2\eta_1}{d}\right) + 5{\cdot}41\right]\frac{d^2}{\eta_1^2}, \tag{3}$$

whilst that for panel B is

$$K_b = \left[2{\cdot}2\left(\frac{2\eta_2 - 2\eta_1}{d - 2\eta_1}\right)^3 + 1{\cdot}61\left(\frac{2\eta_2 - 2\eta_1}{d - 2\eta_1}\right)^{\frac{1}{2}} + 4\right]\left(\frac{1}{1 - 2\eta_1/d}\right)d^2/(\eta_2 - \eta_1)^2, \tag{4}$$

and that for panel C is

$$K_c = \frac{5{\cdot}98\left(2 - \dfrac{2\eta_2}{d}\right)^2}{\left(1 - \dfrac{2\eta_2}{d}\right)^3}\,\frac{d^2}{(d - \eta_2)^2}. \tag{5}$$

Now when the stiffeners are in their optimum position the buckling resistance of all three panels will be approximately equal. In order to find an approximate optimum position we therefore assume that the buckling stresses of the three panels are equal. In the clamped case solving equations (3), (4) and (5) for the three unknowns η_1, η_2, and K the respective values of $0{\cdot}139d$, $0{\cdot}289d$ and 319 are found. The values of η_1 and η_2 are reasonably close to the exact values of $0{\cdot}135d$, $0{\cdot}284d$ and 355 obtained in Reference 1. Having determined the approximate optimum configuration the corresponding value of K can be obtained from equation (11) of Reference 1. This value K_s includes the continuity effect which is ignored in obtaining the approximate optimum configuration, and is the exact buckling stress for the given configuration.

A similar method is used in the case of 3, 4, and 5 stiffeners using (1a) or (2) for the panel at the compression edge, depending on the edge condition, and (1a) for the intermediate panels. Since it is reasonable to assume that the edge condition at the tensile

TABLE 1. OPTIMUM PLACING OF LONGITUDINAL STIFFENERS

Flanges provide clamped support

No. of stiffeners		K	1	2	3	4	5	6	K_s
One	Exact optimum		0·22d	—	—	—	—	—	161
	Approx. optimum	139	0·221d	—	—	—	—	—	161
Two	Exact optimum		0·136d	0·284d	—	—	—	—	356
	Approx. optimum	319	0·139d	0·289d	—	—	—	—	327
Three	Approx. optimum	578	0·101d	0·204d	0·327d	—	—	—	597·5
Four	Approx. optimum	915	0·080d	0·158d	0·246d	0·352d	—	—	925·6
Five	Approx. optimum	1330	0·066d	0·129d	0·198d	0·278d	0·369d	—	1322
Six	Approx. optimum	1822	0·056d	0·109d	0·166d	0·228d	0·298d	0·382d	—

Flanges provide simple support

No. of stiffeners		K	1	2	3	4	5	6	K_s
One	Exact optimum		0·20d	—	—	—	—	—	129
	Approx. optimum	120	0·208d	—	—	—	—	—	—
Two	Exact optimum		0·123d	0·275d	—	—	—	—	313
	Approx. optimum	293	0·129d	0·283d	—	—	—	—	—
Three	Approx. optimum	544	0·093d	0·198d	0·323d	—	—	—	532
Four	Approx. optimum	872	0·073d	0·152d	0·242d	0·349d	—	—	842
Five	Approx. optimum	1277	0·060d	0·124d	0·194d	0·273d	0·367d	—	1220
Six	Approx. optimum	1760	0·051d	0·104d	0·162d	0·225d	0·296d	0·381d	—

TABLE 2. CONVERGENCE TESTS: DETERMINATION OF K_s (SEE TABLE 1)

Longitudinal edges clamped 5 stiffeners $\eta_1 = 0·066d$ $\eta_2 = 0·129d$ $\eta_3 = 0·198d$ $\eta_4 = 0·278d$ $\eta_5 = 0·369d$ $\alpha = 0·065$	Size of determinant $N \times N$	18 × 18	20 × 20	21 × 21	22 × 22	23 × 23	24 × 24	30 × 30	35 × 35
	Buckling coefficient K_s	2557·1	1551·1	1398·5	1346·2	1329·3	1325·3	1323	1322·3
Longitudinal edges simply supported 5 stiffeners $\eta_1 = 0·060d$ $\eta_2 = 0·124d$ $\eta_3 = 0·194d$ $\eta_4 = 0·273d$ $\eta_5 = 0·367d$ $\alpha = 0·065$	Size of determinant $N \times N$	25 × 25	30 × 30	35 × 35					
	Buckling coefficient K_s	1225	1220	1220					
Longitudinal edges clamped 4 stiffeners $\eta_1 = 0·08d$ $\eta_2 = 0·158d$ $\eta_3 = 0·246d$ $\eta_4 = 0·352d$ $\alpha = 0·08$	Size of determinant $N \times N$	18 × 18	24 × 24	30 × 30					
	Buckling coefficient K_s	948·3	927·1	926·2					

edge has little effect on the buckling stress of the tensile panel, (1b) is used for both edge conditions. The results are given in Table 1, where the values given under K are the approximate values of K obtained from the above equations and under K_s are the exact values of K for the given configuration. In determining K_s, 35×35 determinants were used. Table 2 gives details of the convergence tests which were carried out. Comparison of the approximate with the available exact optimum solutions shows that the agreement is quite good.

3. DETERMINATION OF STIFFENER RIGIDITIES

As stated in the Introduction, one case which is of particular interest to the designer, since it will simplify fabrication, is when all the stiffeners have the same flexural rigidity and area. Having determined the approximate optimum placing of the stiffeners by the method described in Section 2, equation (8) of Reference 1 was solved for the particular case where $\gamma_1 = \gamma_2 = \gamma_3 = \ldots = \gamma_p$ and $B_1 = B_2 = B_3 = \ldots = B_p$. Table 3 gives details of the convergence tests conducted for the case of 3, 4 and 5 longitudinal stiffeners.

The results obtained are plotted in Figs. 4, 5 and 6.

TABLE 3. CONVERGENCE TESTS: DETERMINATION OF EQUAL GAMMA'S.

Longitudinal edges simply supported 3 stiffeners $\eta_1 = 0{\cdot}093d$ $\eta_2 = 0{\cdot}198d$ $\eta_3 = 0{\cdot}323d$ $K = 532$, $B = 0{\cdot}1$	Size of determinant $N \times N$		15×15	20×20	25×25	30×30	Size used in calculations
	Value of γ	$\alpha = 0{\cdot}8$	43·19	43·33	43·34	43·35	15×15
		$\alpha = 2{\cdot}4$	309·2	309·3	309·3	309·3	
Longitudinal edges simply supported 4 stiffeners $\eta_1 = 0{\cdot}073d$ $\eta_2 = 0{\cdot}152d$ $\eta_3 = 0{\cdot}242d$ $\eta_4 = 0{\cdot}349d$ $K = 842$, $B = 0{\cdot}1$	Size of determinant $N \times N$		15×15	20×20	25×25	30×30	15×15
	Value of γ	$\alpha = 0{\cdot}8$	64·87	64·91	65·01	65·03	
		$\alpha = 2{\cdot}4$	464·7	464·7	464·7	464·7	
Longitudinal edges clamped 5 stiffeners $\eta_1 = 0{\cdot}066d$ $\eta_2 = 0{\cdot}129d$ $\eta_3 = 0{\cdot}198d$ $\eta_4 = 0{\cdot}278d$ $\eta_5 = 0{\cdot}369d$ $K = 1322$, $B = 0{\cdot}1$	Size of determinant $N \times N$		16×16	20×20	25×25		
	Value of γ	$\alpha = 1{\cdot}6$	288·1	288·2	288·2		16×16

DISCUSSION OF RESULTS

It will be noted from Figs. 4, 5 and 6 and from Table 1 that when two or more longitudinal stiffeners are employed, the influence upon the parameters K and γ of increasing the flange support from that of a simple support to a clamped support is slight for values of b/d less than 1·0.

In order to assist designers approximate relationships have been developed which approximate the theoretical relationships between γ and b/d for values of b/d within the range 0·5 to 1·6. For values of b/d outside this range, it is recommended that the curves in Figs. 4, 5 and 6 be used. The approximate relationships are as follows:

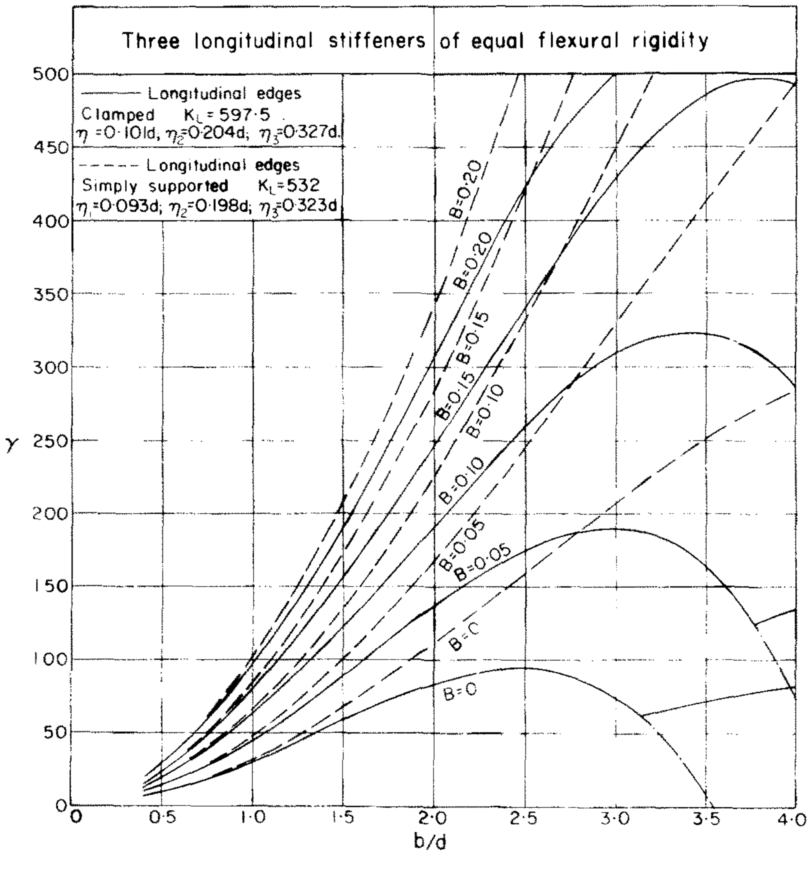

FIG. 4.

Two stiffeners

Longitudinal edges simply supported: $(30{\cdot}69+202{\cdot}7B)\alpha^2-(4{\cdot}87+20{\cdot}69B)\alpha^3$

Longitudinal edges clamped: $(35{\cdot}37+195{\cdot}8B)\alpha^2-(11{\cdot}29+15{\cdot}94B)\alpha^3$

Three Stiffeners

Longitudinal edges simply supported: $(35{\cdot}54+422{\cdot}9B)\alpha^2-(4{\cdot}278+73{\cdot}72B)\alpha^3$

Longitudinal edges clamped: $(37{\cdot}18+423{\cdot}5B)\alpha^2-(7{\cdot}852+87{\cdot}98B)\alpha^3$

Four Stiffeners

Longitudinal edges simply supported: $(46{\cdot}52+687{\cdot}0B)\alpha^2-(5{\cdot}961+98{\cdot}81B)\alpha^3$

Longitudinal edges clamped: $(46{\cdot}50+708{\cdot}5B)\alpha^2-(8{\cdot}888+142{\cdot}9B)\alpha^3$

Five Stiffeners

Longitudinal edges simply supported: $(57{\cdot}58+1010B)\alpha^2-(7{\cdot}420+123{\cdot}4B)\alpha^3$

Longitudinal edges clamped: $(58{\cdot}02+1036B)\alpha^2-(11{\cdot}42+179{\cdot}1B)\alpha^3$

The above laws yield values of γ which are within 5% of the actual values.

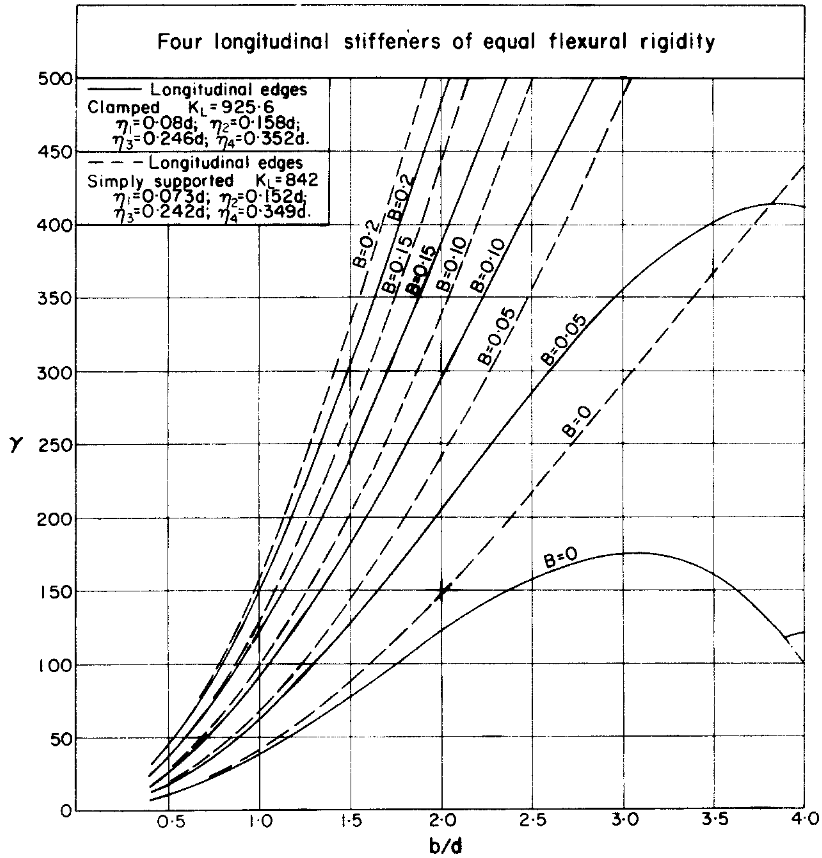

FIG. 5.

With regard to the placing of the stiffeners, it will be noted that the position of the stiffeners has been specified to within one thousandth part of the depth of the girder. In practice this would mean positioning the stiffeners to the nearest $\frac{1}{4}$ in. on a web 20 ft deep; which is quite practical. The width of the stiffener and the attaching weld will however reduce the clear width of the panels to less than that assumed in the theory, with the result that the buckling resistance stated will in fact be less than that obtained in a practical girder. No allowance for this fact has been made in the current analysis.

Now that a solution for the buckling under pure bending of a web reinforced by a number of longitudinal stiffeners has been obtained, it is possible to examine the buckling of such webs when loaded in combined Shear and Bending. When examining this case, the approximate method of determining the optimum position of the stiffeners, which has been shown to be useful in this paper, will clearly help to reduce the computational work involved.

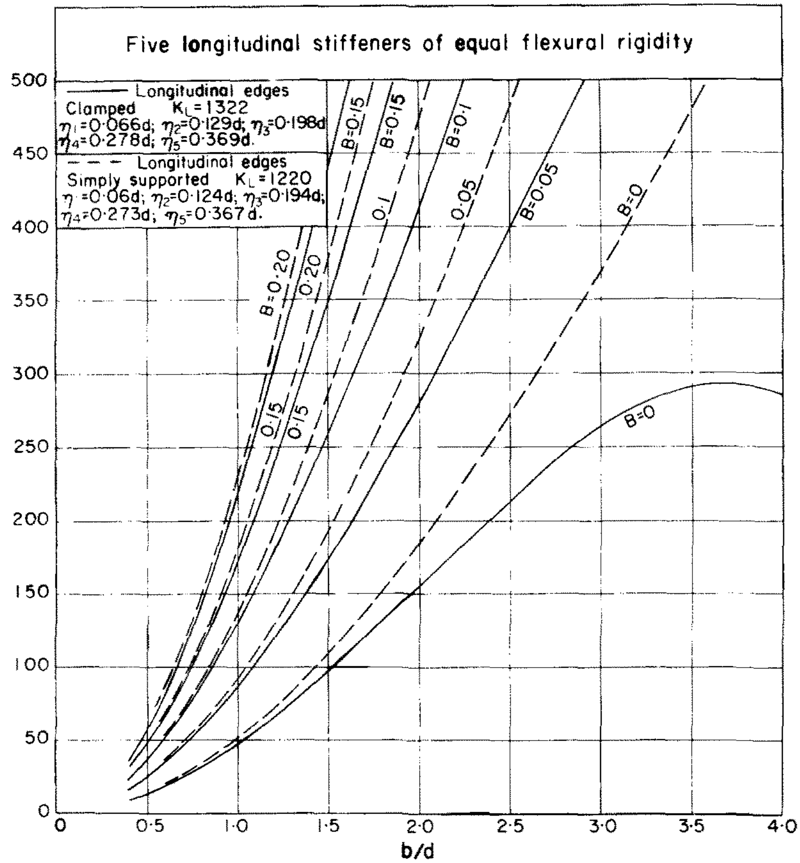

FIG. 6.

CONCLUSION

The approximate optimum placing of 3, 4 or 5 longitudinal stiffeners has been determined. For identical stiffeners placed in these positions, relationships between the flexural rigidity and area of the longitudinal stiffeners and the spacing of the transverse stiffeners have been determined. For values of b/d less than 1·5 the buckling resistance and the rigidity of the longitudinal stiffeners is little affected by an increase in longitudinal restraint from that of a simple support to a clamped support.

Acknowledgement—The results presented in this paper are based on calculations which were carried out on the Elliot Computer of Hull University.

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[2] K. C. ROCKEY and D. M. A. LEGGETT, *Proc. Inst. Civ. Engrs* **21**, 161 (1962).
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Zusammenfassung—Der Beitrag bietet eine Lösung für die Knickung eines Stegträgers unter reinen Biegungsverhältnissen, wobei der Träger mit 3, 4 oder 5 längsweisen Aussteifungsstücken verstärkt wird. Die vorgeschlagenen Lösungen beziehen sich auf Fälle, bei denen vorausgesetzt wird, dass die beiden längsweisen Kanten entweder einfach gestützt, oder steif angeklemmt sind. In beiden Fällen wird angenommen, dass längs der Transversalaussteifungen eine einfache Stützbedingung besteht.

Die optimale Lage dieser Aussteifungsstücke wird besprochen, und die zahlenmässigen Beziehungen zwischen γ and B, den Parametern der Aussteifungsstücke, und α, dem Seitenverhältnis, werden angegeben, und zwar für einen Fall, bei dem alle längsweisen Aussteifungen identisch sind.

Абстракт—Работа дает решения для выпучивания при простом изгибе стенок сплошных балок, усиленных посредством 3, 4, или 5 продольных элементов жесткости. Эти решения предложены для случаев, где оба продольные края или имеют простую опору или же жестко закреплены. В обоих случаях предполагается, что условия простой опоры существуют вдоль поперечных элементов жесткости.

Дискуссируется оптимальное расположение этих элементов жесткости и даются численные отношения между параметрами γ и B элементов жесткости и относительным удлиннением α для случая, где все продольные элементы одинаковы.